17. Oscillations

17.1 Simple harmonic oscillations

Paper 4

Marking Scheme

Q1.

(b)(iii)	frequency = 1 / 0.25 = 4.0 Hz	**A1**

Q2.

(a)	straight line through the origin shows that *a* is proportional to *x*	**B1**
	negative gradient shows that *a* and *x* are (always) in opposite directions	**B1**
(b)(i)	$a = -\omega^2 x$ $\omega = \sqrt{(2A / 3Y)}$	**A1**
(b)(ii)	$v_0 = \omega x_0$	**C1**
	$= 3Y \times \sqrt{(2A / 3Y)}$ $= \sqrt{(6AY)}$	**A1**
(c)	$\omega = 2\pi / T$ $(= 2\pi / 0.75)$	**C1**
	$x = 1.8 \sin (8.4\, t)$	**A1**

Q3.

(a)	(motion in which) acceleration is (directly) proportional to displacement	**B1**
	(motion in which): acceleration is (always) in the opposite direction to displacement **or** acceleration is (always) directed towards a fixed point	**B1**
(b)(i)	amplitude = (9.5 – 3.5) / 2 = 3.0 cm	**A1**
(b)(ii)	$\omega = v_0 / x_0$	**C1**
	$= 9.5 / 3.0 = 3.2$ rad s^{-1}	**A1**
(b)(iii)	$T = 2\pi / \omega$	**C1**
	$= 2\pi / 3.2$ = 2.0 s	**A1**
(b)(iv)	attempted sinusoidal curve starting with a minimum at $t = 0$	**B1**
	sinusoidal curve of period 2.0 s from $t = 0$ to $t = 6.0$ s	**B1**
	all peaks shown at $h = 9.5$ cm	**B1**
	all troughs shown at $h = 3.5$ cm	**B1**

Q4.

(a)	arrow from sphere, perpendicular to string, pointing left and down	**B1**
(b)(i)	amplitude = 0.016 m	**A1**
(b)(ii)	angular frequency = $2\pi / T$	**C1**
	$= 2\pi / 0.40$ = 16 rad s^{-1}	**A1**

Q5.

(a)(i)	$\omega = 2\pi f$	**C1**
	$f = 9.7 / 2\pi$ $= 1.5$ Hz	**A1**
(a)(ii)	amplitude $= \sqrt{(11.6)} = 3.4$ cm	**A1**
(a)(iii)	$a_0 = \omega^2 x_0$	**C1**
	$= 9.7^2 \times 3.4 \times 10^{-2}$ $= 3.2$ m s^{-2}	**A1**
(b)	sketch: straight line through the origin with negative gradient	**B1**
	line with negative gradient passing through $(+3.4, -a_0)$ and $(-3.4, +a_0)$	**B1**
	line with ends at $x = \pm 3.4$ cm and $a = \pm a_0$	**B1**

Q6.

(a)	(motion in which) acceleration is (directly) proportional to displacement	**B1**
	(motion in which): acceleration is (always) in the opposite direction to displacement **or** acceleration is (always) directed towards a fixed point	**B1**
(b)(i)	$\omega = 2\pi / T$	**C1**
	$\omega = 2\pi / 3.0$ $= 2.1$ rad s^{-1}	**A1**

Q7.

(a)(i)	amplitude $= ½ \times 7.2 \times 10^{-15}$ $= 3.6 \times 10^{-15}$ m	**A1**
(a)(ii)	$\omega = 2\pi / (0.20 \times 10^{-6})$ $= 3.1 \times 10^7$ rad s^{-1}	**A1**
(a)(iii)	$v_0 = \omega x_0$	**C1**
	$v_0 = 3.1 \times 10^7 \times 3.6 \times 10^{-15} = 1.1 \times 10^{-7}$ m s^{-1}	**A1**

Q8.

(a)	straight line through origin shows that a is proportional to x	**B1**
	negative gradient shows that a is in opposite direction to x	**B1**
(b)(i)	$a_0 = \omega^2 x_0$ **or** $a = -\omega^2 x$ **or** $\omega^2 =$ – gradient	**C1**
	$\omega = \sqrt{(0.40 / 0.050)}$ $= 2.8$ rad s^{-1}	**A1**
(b)(ii)	$k = \omega^2 L$ $= 2.8^2 \times 1.24$	**C1**
	$= 9.7$ m s^{-2}	**A1**

Q9.

(b)(i)	$T = 2\pi / \omega$	**C1**
	$= 2\pi / 5.0\pi$ $= 0.40$ s	**A1**
(b)(ii)	displacement scale labelled –1.0, –0.5, (0), 0.5, 1.0 on the 2 cm tick marks	**B1**
	t scale labelled 0.2, 0.4, 0.6, 0.8, 1.0, 1.2 on the 2 cm tick marks	**B1**
(b)(iii)	$\phi = 2\pi\Delta t / T$ $= 2\pi \times 0.10 / 0.40$ **or** $2\pi \times 0.30 / 0.40$	**C1**
	$= 1.6$ rad **or** 4.7 rad	**A1**

Q10.

(a)	$a = -\omega^2 x$	**M1**
	a = acceleration, x = displacement from equilibrium position and ω = angular frequency	**A1**
(b)(i)	$x_0 = 0.12$ m	**A1**
(b)(ii)	$v = \omega\sqrt{(x_0^2 - x^2)}$ two (x, v) pairs correctly read from Fig. 3.2 (one may be (x_0, 0) or value of x_0 from **(i)**)	**C1**
	e.g. $0.20 = \omega\sqrt{(0.12^2 - 0)}$ leading to $\omega = 1.7$ rad s^{-1}	**A1**
(c)(ii)	closed loop surrounding the origin with maximum x at ± 0.060 m passing through $v = 0$	**B1**
	maximum velocity shown as ± 0.10 m s^{-1} passing through $x = 0$	**B1**

Q11.

(a)(i)	$x_0 = 8.0$ cm	**A1**
(a)(ii)	$\omega = 2\pi / T$	**C1**
	$= 2\pi / 4.0 = 1.6$ rad s^{-1}	**A1**

Q12.

(a)	acceleration (directly) proportional to displacement	**B1**
	acceleration is in opposite direction to displacement	**B1**
(b)	$\omega^2 = 2k / m$ **and** $\omega = 2\pi f$	**C1**
	$(2\pi f)^2 = (2 \times 130) / 0.84$	**C1**
	$f = 2.8$ Hz	**A1**

Q13.

(a)	acceleration in opposite direction to displacement shown by – sign	**B1**
	g / L is constant	**M1**
	(so) acceleration is (directly) proportional to displacement	**A1**
(b)	$\omega^2 = g / L$	**C1**
	$\omega = 2\pi / T$ **or** $\omega = 2\pi f$ **and** $f = 1 / T$	**C1**
	$(2\pi / T)^2 = 9.81 / 0.18$ $T = 0.85$ s	**A1**

Q14.

(a)(i)	5.0 cm	**A1**
(a)(ii)	$\omega = 2\pi / T$ **or** $\omega = 2\pi f$ **and** $f = 1 / T$	**C1**
	$\omega = 2\pi / 4.0$ $= 1.6$ rad s^{-1}	**A1**
(a)(iii)	$v_0 = \omega x_0$	**C1**
	$= 1.57 \times 5.0$ $= 7.9$ cm s^{-1}	**A1**
(b)	• initial pull was to the right • distance from X to trolley (at equilibrium) is 20 cm • period is 4.0 s • initial motion undamped • motion becomes damped at/from 12 s • damping is light • maximum speed at 1 s, 3 s, etc. / stationary at 2 s, 4 s, etc. *Any three points, 1 mark each*	**B3**
(c)	sketch: closed loop encircling (20, 0)	**B1**
	minimum L shown as 15 cm and maximum L shown as 25 cm	**B1**
	minimum v shown as –7.9 cm s^{-1} and maximum v shown as +7.9 cm s^{-1}	**B1**

Q15.

(a)	straight line through the origin	**B1**
	negative gradient	**B1**
(b)	$a = (-)\omega^2 x$ **and** $T = 2\pi / \omega$	**C1**
	e.g. $\omega = \sqrt{(0.80 / 0.12)}$ (*any correct pair of values of a and x*) (= 2.58 rad s^{-1})	**C1**
	$T = 2\pi / 2.58$ $= 2.4$ s	**A1**
(c)(i)	Point labelled P at one end of the line	**B1**
(c)(ii)	Point labelled Q at displacement with magnitude more than half but less than maximum	**B1**

Q16.

(a)(i)	amplitude = 4.9 cm	**A1**
(a)(ii)	frequency = 2700 / 60 = 45 Hz	**A1**
(a)(iii)	$v_0 = x_0\omega$ **and** $\omega = 2\pi f$	**C1**
	$v_0 = 4.9 \times 10^{-2} \times 2\pi \times 45$ $= 14$ m s^{-1}	**A1**
(a)(iv)	$v = \omega (x_0^2 - x^2)^{½}$ $= 2\pi \times 45 \times [(4.9 \times 10^{-2})^2 - (2.6 \times 10^{-2})^2]^{½}$	**C1**
	$= 12$ m s^{-1}	**A1**
(b)	$F = ma$ **and** $a_0 = v_0\omega$ **or** $a_0 = x_0\omega^2$	**C1**
	$F = 0.64 \times 13.9 \times 2\pi \times 45$ **or** $0.64 \times 4.9 \times (2\pi \times 45)^2$	**C1**
	$= 2500$ N	**A1**

Q17.

(a)	($\omega = 2\pi / T$ and $T = 2.2$ s so) $\omega = 2\pi / 2.2 = 2.9$ rad s^{-1}	**A1**
(b)(i)	$\omega^2 = g / R$	**C1**
	$R = 9.81 / 2.86^2$ $= 1.2$ m	**A1**
(b)(ii)	$v_0 = \omega x_0$	**C1**
	$= 2.9 \times 3.0 \times 10^{-2}$ $= 0.087$ m s^{-1}	**A1**

Q18.

(a)	acceleration (directly) proportional to displacement	**B1**
	acceleration in opposite direction to displacement **or** acceleration (directed) towards equilibrium position	**B1**
(b)	$v = \omega(x_0^2 - x^2)^{½}$ **and** $\omega = 2\pi f$ **or** $v_0 = x_0\omega$ **and** $\omega = 2\pi f$	**C1**
	substitution of any correct point from graph, e.g. for $x = 0$: $0.25 = 2\pi f \times 8.8 \times 10^{-2}$	**C1**
	$f = 0.45$ Hz	**A1**
(c)	$1 / 0.45 = 2\pi \times (L / 9.81)^{½}$	**C1**
	$L = 1.2$ m	**A1**
(d)	ellipse about the origin with same intercepts on x-axis	**B1**
	ellipse about the origin crossing v-axis inside original loop	**B1**

Q19.

(a)	acceleration (directly) proportional to displacement	**B1**
	acceleration is in opposite direction to displacement **or** acceleration is (directed) towards a fixed point	**B1**
(c)	$T = 2\pi / \omega$	**C1**
	$2\pi / 3.3 = 2\pi \times (L / 9.81)^{½}$	**C1**
	$L = 0.90$ m	**A1**

Q20.

(a)(i)	amplitude = 0.020 m	**A1**
(a)(ii)	$T = 0.60$ s	**C1**
	$f = 1 / T$ $= 1.7$ Hz	**A1**
(a)(iii)	$a = (-)\omega^2 x$ **and** [$\omega = 2\pi f$ **or** $\omega = 2\pi / T$]	**C1**
	$a = (4\pi^2 / 0.60^2) \times 2.0 \times 10^{-2}$ $= 2.2$ m s^{-2}	**A1**
(b)	$1.67 = (1 / 2\pi) \times [(24 \times 10^{-4} \times \rho \times 9.81) / 0.23]^{1/2}$	**C1**
	$\rho = 1.1 \times 10^3$ kg m^{-3}	**A1**

Q21.

(a)(i)	0.10 s **or** 0.30 s **or** 0.50 s **or** 0.70 s **or** 0.90 s	**A1**
(a)(ii)	0 **or** 0.40 s **or** 0.80 s	**A1**
(b)(i)	$\omega = 2\pi / T$	**C1**
	$= 2\pi / 0.40$ $= 16 \text{ rad s}^{-1}$	**A1**
(b)(ii)	$v_0 = \omega x_0$	**C1**
	$= 15.7 \times 2.5 \times 10^{-2}$ $= 0.39 \text{ m s}^{-1}$	**A1**
	or	
	tangent drawn at steepest part and working to show attempted calculation of gradient	**(C1)**
	leading to $v_0 = 0.39 \text{ m s}^{-1}$ (*allow ± 0.15 m s*$^{-1}$)	**(A1)**
(b)(iii)	$a_0 = \omega^2 x_0$	**C1**
	$a_0 = (15.7^2 \times 2.5 \times 10^{-2})$ $= 6.2 \text{ m s}^{-2}$	**A1**
	or	
	$a_0 = \omega v_0$	**(C1)**
	$a_0 = 15.7 \times 0.39$ $= 6.2 \text{ m s}^{-2}$	**(A1)**
(c)	period is shorter/lower	**B1**
	Any one from: • greater spring constant/stiffness • (restoring) force is greater (for any given extension) • acceleration is greater (for any given extension) • greater energy/maximum speed (for a given amplitude)	**B1**

Q22.

(a)(i)	distance from a (reference) point in a given direction	**B1**
(a)(ii)	line is not straight **or** gradient is not constant	**B1**
(b)(i)	0.85–0.90 cm	**A1**
(b)(ii)	$a = -(2\pi f)^2 x$	**C1**
	e.g. $1.2 = 4\pi^2 \times f^2 \times (0.90 \times 10^{-2})$	**C1**
	$f = 1.8 \text{ Hz}$	**A1**
(c)	complete circle/ellipse enclosing the origin	**B1**
	closed shape passing through $(0, \pm v_0)$ and $(\pm x_0, 0)$	**B1**

Q23.

(b)(i)	$\omega = 2\pi / T$	**C1**
	$T = 0.80$ s, so $\omega = 2\pi / 0.80$ $\omega = 7.9 \text{ rad s}^{-1}$	**A1**
(b)(ii)	$\omega^2 = 2k / M$	**C1**
	$7.9^2 = 2k / 1.2$ $k = 37 \text{ N m}^{-1}$	**A1**

Q24.

(a)(i)	straight line through origin indicates acceleration $\propto$ displacement	**B1**
	negative gradient shows acceleration and displacement are in opposite directions	**B1**
(a)(ii)	$a = -\omega^2 y$ **and** $\omega = 2\pi f$ $4.5 = (2\pi \times f)^2 \times 8.0 \times 10^{-3}$ *(or other valid read-off)*	**C1**
	$f = 3.8$ Hz	**A1**
(b)(i)	maximum displacement upwards/above rest/above the equilibrium position	**B1**
(b)(ii)	(just leaves plate when) acceleration = $9.81\,\text{m s}^{-2}$	**C1**
	$9.81 = (2\pi \times 3.8)^2 \times y_0$ **or** $9.81 = 563 \times y_0$	**C1**
	amplitude = 17 mm	**A1**

Q25.

(a)	acceleration proportional to displacement	**B1**
	acceleration directed towards fixed point **or** displacement and acceleration in opposite directions	**B1**
(b)(ii)1.	$\omega^2 = 2k/m$	**C1**
	$= (2 \times 230) / 0.950$	**C1**
	$\omega = 22\,\text{rad s}^{-1}$	**A1**
(b)(ii)2.	$T = 2\pi / \omega$	**C1**
	$T = (2\pi / 22) = 0.286$ s time $= 1.5T$ $= 0.43$ s	**A1**

Q26.

(a)	$\omega^2 = 2g / L$	**C1**
	$T = 2\pi / \omega$	**C1**
	$\omega^2 = (2 \times 9.81) / 0.19$ $\omega = 10.2\,(\text{rad s}^{-1})$ $T = 2\pi / 10.2$ $= 0.62$ s	**A1**

Q27.

(a)	(defining equation of s.h.m. is) $a = -kx$ where k is a constant **or** $a \propto -x$	**B1**
	g and L are constant (so $a \propto -x$ and hence s.h.m.)	**B1**
(b)	$T = 0.50$ s **and** $T = 2\pi / \omega$	**C1**
	$\omega^2 = 2g / L$	**C1**
	$L = (2 \times 9.81 \times 0.50^2) / 4\pi^2$ $= 0.12$ m	**A1**